

# Are gluons massive ?

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## Abstract

It is claimed that only one gluon is massless and the other seven gluons are massive. Out of eight gluons, six are colored and two are neutral. Among neutral gluons, one is massless and other one is massive. Massive neutral gluon is heavier than the colored gluons. Gluons can only be predicted by set theory but not by  $SU(3)$ .

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Recently a strong indication for a deviation from the standard model (SM) has been obtained by PIBETA Collaboration [1] in the  $\pi^+ \rightarrow e^+ \nu \gamma$  decay. A brief discussion on this observation is given in Ref. [2]. A variety of literature is available which give the indication of New Physics [3]. This motivates the possibility that there may be some physical phenomenon which did not describe correctly and needs revision.

The SM is an incomplete theory: some kind of new physics is required in order to understand the patterns of quark and lepton masses and mixings, and generally to understand flavor dynamics [4]. There are strong theoretical arguments suggesting that new physics cannot be far from the electroweak scale. Therefore, the determination of Cabbibo-Kobayashi-Maskawa (CKM) matrix [5, 6] that parametrizes the weak charged current interactions of quarks is currently a central theme in particle physics. Test of its structure, conveniently represented by the unitarity triangle, have to be performed; they

will allow a precision determination of the SM contributions to various observables and possibly reveal the onset of new physics contributions. Indeed, the four parameters of this matrix govern all flavour changing transitions involving quarks in the SM. These include tree level decays mediated by  $W$  bosons, which are essentially unaffected by new physics contributions. The flavour changing neutral current (FCNC) transitions responsible for rare and CP violating decays in the SM, which involve gluons, are sensitive probes of new physics.

The Standard Model (SM), all matter is made up of three kinds of elementary particles: leptons, quarks, and gauge bosons (mediators). There are six leptons with three generations. Similarly, there are six quarks, each quark and antiquark comes in three color charges (red, green, blue), so there are 36 of them [7]. Before the Glashow-Salam-Weinberg (GSW) theory, there were four fundamental forces in nature: strong, weak, electromagnetic, and gravitational. The GSW model treats weak and electromagnetic interactions as different manifestation of a single electroweak force, and in this sense the four forces reduce to three. Each of these forces is mediated by the exchange of particle. The gravitational force is mediated by Graviton, electroweak force by photon and intermediate vector bosons, and strong force by gluons; and the Casimir force<sup>1</sup>.

Force	Charge	Mediators (gauge bosons)
Casimir	$\{ \}$	?
Gravitational	$\{0\}$	Graviton
Electroweak	$\{+, -\}$	$\{\text{Photon}, W^+, W^-, Z^0\}$
Strong	red, green, blue	eight colored and massless gluons [7]

If we look upon the above table, the mediators of their respective forces has a certain relation which we can describe by set theory as discussed in appendix A.

Charges	No. of mediators	Set of charges	Subsets of mediators
0	$2^0 = 1$	$\{ \}$	Casimirion
1	$2^1 = 2$	$\{0\}$	$\mathfrak{g}, \mathcal{G}$
2	$2^2 = 4$	$\{+, -\}$	$\gamma, W^+, W^-, Z^0$
3	$2^3 = 8$	$\{r, g, b\}$	$g_0, g_r, g_g, g_b, g_{rg}, g_{rb}, g_{gb}, g_{rgb}$

where  $r, g, b$  stands for red, green, blue respectively i.e. the color charges. We

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<sup>1</sup>The Casimir force also exist in nature which is the manifestation of zero-point energy [8].

restrict here to the discussion of strong force while the discussion on Casimir and gravitational forces are out of scope of this article<sup>2</sup>. The mediators  $g_0, g_r, g_g, g_b, g_{rg}, g_{rb}, g_{gb}, g_{rgb}$  are the gluons for the strong force. We claim that only one gluon i.e.  $g_0$  which associates to empty subset is massless as photon ( $\gamma$ ) is associated to empty subset in the case of electroweak mediators. The remaining gluons  $g_r, g_g, g_b, g_{rg}, g_{rb}, g_{gb}, g_{rgb}$  are massive as the intermediate vector bosons  $W^+, W^-, Z^0$ . The gluons  $g_0$  and  $g_{rgb} (\equiv G_0)$  are neutral. The gluons  $g_{rg}, g_{rb}, g_{gb}$  are respectively equivalent to  $g_{\bar{b}}, g_{\bar{g}}, g_{\bar{r}}$  as

$$\begin{aligned} \bar{r} &\equiv gb, & r &\equiv \bar{g}\bar{b}, \\ \bar{g} &\equiv rb, & g &\equiv \bar{r}\bar{b}, \\ \bar{b} &\equiv rg, & b &\equiv \bar{r}\bar{g}. \end{aligned} \tag{1}$$

Let us give an example to support our argument, on page 280 of Ref. [7], a red quark turned into a blue quark, emitting a red-antiblue gluon. Let us concentrate on the charge of the gluon in the above example. The charge antiblue,  $\bar{b} = rg$ , is composed of ‘red and green’, while the charge over the gluon is ‘red-antiblue’ equivalent to ‘red-red-green’ which does not obey the group property because ‘red’ is repeated twice, as we define the anticolors in Eq. (1). This argues that a gluon will never carry a charge like  $r\bar{b}$  etc. It will only carry a color or anti-color. We claim that in the above example a red quark will turn into blue quark, emitting a green gluon. We also point out that

$$\begin{aligned} m_{g_r} &= m_{g_{gb}} (= m_{g_{\bar{r}}}) \\ m_{g_g} &= m_{g_{rb}} (= m_{g_{\bar{g}}}) \\ m_{g_b} &= m_{g_{rg}} (= m_{g_{\bar{b}}}) \end{aligned}$$

The gluon  $G_0$  will be massive than the gluons  $g_r, g_g, g_b, g_{\bar{b}}, g_{\bar{g}}, g_{\bar{r}}$  as the neutral

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<sup>2</sup>As Casimir force is the quantum fluctuation of the vacuum and hence, no charge exist there. We give an empty set to its charge. So, only one massless mediator is required for such interactions. We suggest its name as Casimirion. Now, we come to the gravitational force. All the matter has charge neutral, we cannot associate an empty set to the charge upon the matter. Therefore, we assign a ‘0’ charge upon the matter. This set has two subsets and both are improper subsets (see appendix A). This means that there must be two mediators to describe gravitational interactions. As nature never leave the gaps unfilled. So, we predict that there must be two neutral gravitons, one massless  $\mathbf{g}$  and other massive  $\mathcal{G}$ .

vector boson  $Z^0$  is massive to charged vector boson  $W^\pm$ . That is

$$m_{G_0} > m_{g_i}, \quad i = r, g, b, \bar{b}, \bar{g}, \bar{r}$$

We conclude that one gluon is massless and the seven gluons are massive. Of them massless gluon is neutral and one massive gluon is color singlet. The rest of the six gluons are colored and massive, their mass relations are also given.

The massive gluons can interact with each other in similar way as the vector bosons interact, while the massless gluon play the similar role as photon. We can divide QCD in two branches, one which deals the interaction mediated by massless gluon and the other which deals with the interactions by the rest of gluons, say chromo-magnetic and chromo-weak, respectively. The names suggested on the basis of electro-magnetic and weak theory. The marriage of chromo-magnetic and chromo-weak theories will result in QCD. The chromo-magnetic and chromo-weak interactions will collectively be called as strong interactions. We summarize our findings as:

- color-induced interactions between quarks are mediated by gluons and electroweakly neutral spin-1 particles,
- only one gluon is massless and remaining seven gluons are massive,
- six gluons are colored, three carry color charge and three carry anticolor charge,
- two gluons are color neutral, one massless and one massive,
- neutral massive gluon is heavier than the colored gluons,
- the colored quarks emit and absorb massless gluon in the same way as the electrically charged particles emit and absorb photons,
- the colored quarks emit and absorb massive colored and neutral gluons in the same way as the electrically charged particles emit and absorb vector bosons  $W^\pm$  and  $Z^0$  respectively,
- the gluons can be predicted by set theory but not by  $SU(3)$  in analogy to electroweak mediators  $\gamma, W^+, W^-, Z^0$ .

## A Sets and their subsets

In the late nineteenth century Georg Cantor [9] was the first to realize the potential usefulness of investigating properties of sets in general as distinct from properties of the elements that comprise them [10]. All objects can be defined in terms of sets.

The words set and element are undefined terms of set theory just as sentence, true and false are undefined terms of logic. The founder of set theory, George Cantor, suggested imagining a set as a “collection of definite and separate objects of our intuition or thought. These objects are called elements”. A set is completely determined by its elements; the order in which the elements are listed is irrelevant.

Sets	No. of subsets	Subsets
$\{ \}$	$2^0 = 1$	$\{ \}$
$\{0\}$	$2^1 = 2$	$\{ \}, \{0\}$
$\{+, -\}$	$2^2 = 4$	$\{ \}, \{+\}, \{-\}, \{+, -\}$
$\{r, g, b\}$	$2^3 = 8$	$\{ \}, \{r\}, \{g\}, \{b\},$ $\{r, g\}, \{r, b\}, \{g, b\}, \{r, g, b\}$

A basic relation between sets is that of subset. There are two types of subsets i.e. proper and improper subset. The empty set and set itself are improper subsets of a set. To check whether one finite set is a subset of a given set. If any element of a set is not found to equal any element of the given set. Then, the set is not a subset of the given set.

Hofstadter points out that when you start a mathematical argument with if, let, or suppose, you are stepping in a fantasy world where not only are all the facts of the real world true but whatever you are supposing is also true [11]. Once you are in that world, you can suppose something else. That sends you in subfantasy world where not only is everything in the fantasy world true but also the newthings you are supposing. Of course you can continue stepping into new subfantasy worlds in this way indefinitely. You return one level closer to the real world each time you derive a conclusion that makes a whole if – then or universal statement true. Your aim in a proof is to continue deriving such conclusions until you return to the world from which you made your first supposition. So, in Hofstadter’s terms, the author

invites the reader to enter in fantasy world where one statement is known to be true and the other to prove in this fantasy world.

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## References

- [1] E. Frlež et al. (PIBETA Collaboration), hep-ex/0312029.
- [2] M. V. Chizhov, Discovery of New Physics in Radiative Pion Decays? [hep-ph/0402105 v1]
- [3] G. Altarelli, The standard model electroweak theory and beyond, talk given at the PSI Zuoz Summer School on Phenomenology of Gauge Interactions, 13–19 August, 2000, CERN-TH/2000-291 [hep-ph/0011078]

- V. Pagé, and D. London, CP violation in  $B \rightarrow \rho\pi$ : New Physics signals, [hep-ph/0312025]
- A. J. Buras, Waiting for clear signals of new physics in  $B$  and  $K$  decays, [hep-ph/0402191]
- [4] M. Battaglia et al., The CKM matrix and the unitarity triangle, CERN-2003-002-corr (2003) [hep-ph/0304132]
  - [5] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963)
  - [6] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)
  - [7] D. Griffiths, Introduction to Elementary Particles, John Wiley & Sons (1987)
  - [8] K. A. Milton, The Casimir Effect – Physical manifestation of zero-point energy, World Scientific (2001)
  - [9] Bell, E.T. Men of Mathematics. New York: Simon and Schuster, Inc., 1937.
  - [10] S. S. Epp, Discrete Mathematics with Applications, IInd Edition, 1995 by PWS Publishing Company
  - [11] Douglas Hofstadter, ‘Gödel, Escher, and Bach—an Eternal Golden Braid’, 1979
  - [12] G. Altarelli, Standard Electroweak Model and Beyond, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan
  - [13] P. Hoodbhoy, Proton spin in QCD, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan
  - [14] A. Ali, and A. Ya. Parkhomenko, Branching ratios for  $B \rightarrow K^*\gamma$  and  $B \rightarrow \rho\gamma$  decays in next-to-leading order in the large energy effective theory, Eur. Phys. J. C23 (2002) 89 [hep-ph/0105302]; and private communication
  - [15] A.S. Safir, Eur.Phys.J.direct C3 (2001) 15 [hep-ph/0109232]

- [16] A. Ali, CP-Violation and B Physics, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan;
- D. Denegri, Physics at LHC, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan;
- G. Rolandi, Detectors for High energy Physics, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan;
- C. Salgado, Quark-Gluon Plasma, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan;
- J. Schukraft, ALICE: Detectors and its Physics, 3rd Workshop on Particle Physics, 8–13 March 2004, Islamabad, Pakistan